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Letter to editors

Guy Fayolle · Roudolf Iasnogorodski

This letter is to let you know that we found serious errors in the paper [1], recently published in QUESTA. As we explain in the forthcoming sections, the fundamental error originates in [2] (reference [14] of [1]). For the same reasons, it appears that the main results of [3, Sect. 4, Proposition 3] are also false.

1 The solutions of the boundary value problems in [1–3] are incorrect

The preprint [2] was available in May 2013, but seems now to have been withdrawn from the homepage of the authors, although its content might coincide with [4]. So, for the sake of consistency, we make three subsections, the first one being intended for the readers who are provided with the original text of [2].

1.1 About paper [2]

The crucial part is Sect. 3.2, entitled *Alternative Riemann-Hilbert problem*. The main result is stated on p. 19. The authors aim at transforming the boundary value problem (BVP) (3) into the so-called Problem 3, Eq. (5), which is a basic Riemann BVP.

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G. Fayolle (✉)
INRIA - Paris-Rocquencourt, Domaine de Voluceau, B.P.105, 78153 Le Chesnay Cedex, France
e-mail: Guy.Fayolle@inria.fr

R. Iasnogorodski
ITMO, Lomonossova 9, Saint Petersburg, Russia
e-mail: iasnogorodski@mail.ru

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To this end, they propose to construct an *exterior* function by means of the classical Schwarz reflection principle on the underlying Riemann surface. However, this construction is incorrect, as explained below.

1. Let us first remark that the functions $Y^*(x)$, $Y_*(x)$ correspond, respectively, to $Y_0(x)$, $Y_1(x)$ of [5].
2. Proposition 4, which defines $G_Y(y)$, is somehow redundant with respect to Proposition 3, since we have $Y^*(\mathbf{D}_X \backslash [x_1, x_2]) = \mathbf{D}_Y \backslash [y_1, y_2]$.
3. Proposition 1 is correct, but Lemma 5 is wrong (see the book [5], chap. 5, p. 119).
4. In Theorem 1, Eq. (24), $\mathcal{F}_Y^e(y)$, which coincides with $\mathcal{G}_Y(y)$ defined on p. 23, must be analytic in $\mathbb{C} \backslash \mathbf{D}_Y$. But, this is not true. In fact, in constructing $\mathcal{G}_Y(y)$, the authors take into account the cut $[y_1, y_2]$, but, in a rather strange way, they forget *the second cut* $[y_3, y_4]$, which lies outside $\mathbf{D}_Y$. Hence, even if $\mathcal{G}_Y(y)$ is analytic in $Y_*(\mathbf{D}_X \backslash [x_1, x_2])$, it definitely cannot be analytically continued to the whole domain $\mathbb{C} \backslash \mathbf{D}_Y$. In this respect, we note that working on the Riemann surfaces $\mathbb{S}_X$ and $\mathbb{S}_Y$ (with the local Schwarz reflection principle) is of no use.

Had the results of Sect. 3.2 been correct, this might have produced a substantial simplification for computing the final formula. Alas, in the case of curves of genus 1, homologous to the torus, there is no miracle.

1.2 About paper [1]

Here, the goal is to reduce the determination of the function $P(x, 0)$ to the BVP (6). This is supposed to be done in Sect. 4.3, by establishing Eq. (26). Since the proof explicitly relies on the results of reference [2] (as mentioned on p. 250 in [1]), it is clearly incorrect. Indeed, the function $\mathsf{P}_x^e(x)$ is not analytic in $\mathbb{C} \backslash \mathbf{D}_X$, because of the cut $[x_1, x_2]$. This fatal error cannot be repaired, except by proceeding as in [5]. Just for the sake of completeness, let us say that the solution of (26), *when* $\mathsf{P}_x^e(x)$ *is analytic in* $\mathbb{C} \backslash \mathbf{D}_X$, is correctly given by formula (27).

1.3 About paper [3]

With the same causes producing the same effects, Proposition 3 in Sect. 4 of [3] is also wrong.

2 Conclusion

The general theory to solve the functional equations under consideration has been done in the book [5]. The method proceeds by reduction to a so-called *Carleman problem* (with the associated automorphism $\alpha(t) = \bar{t}$, $t \in \partial \mathbf{D}_X$), which does obviously include the formulation given in Eq. (4).

It is worth pointing out that this kind of functional equation of two complex variables has been recently encountered in various scientific areas, such as analytic combinatorics, statistical physics, etc.

References

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